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DIFFRACTION OF ACOUSTOELECTRIC WAVES BY TUNNEL CAVITIES IN AN UNBOUNDED PIEZOCERAMIC MEDIUM†

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The steady wave process in a piezoceramic space with tunnel cavity-openings under plane deformation conditions in a plane parallel to the axis of symmetry of the material is investigated. The corresponding two-dimensional boundary-value problem of electroelasticity is reduced to a system of three singular integral equations of the second kind. The results of a numerical implementation of the algorithm illustrate the effect of the configuration of the openings, the type and frequency of the excitation and the effect of the connectedness of the mechanical and electric fields on the stress concentration.

Plane and antiplane problems of pulsed and harmonic excitation of piezoelectric media with tunnel cracks or cavities have been investigated, for example, in [1–3]. The system of boundary integral equations of plane problems of electroelasticity for the steady oscillations of solids with smooth boundaries is formulated in [4].

1. We will consider, with respect to Cartesian rectilinear axes x_1, x_2, x_3, a piezoceramic medium, weakened by tunnel cavities along the x_2 axis, the transverse sections of which are bounded by the contours $\Gamma_m (m = 1, 2, \ldots, n)$. We will agree to assume that the x_3 axis coincides with the direction of the electric field lines of preliminary polarization of the ceramics. We will assume that the surfaces of the cavities are free from mechanical loads, and that plane monochromatic waves of the appropriate types are incident on them from infinity.

We will assume that the curvatures of the contours Γ_m satisfy a Hölder conditions [5] on $\Gamma = \cup \Gamma_m$ and, in addition, $\cap \Gamma_m = \varnothing$.

In this formulation, a state of plane deformation in x_1Ox_3 occurs in a medium with cavities. The complete system of equations has the following form [6]: the equations of motion of the medium

$$\partial_k \sigma_{ik} = \rho \partial^2 u_i / \partial t^2, \quad \partial_k = \partial / \partial x_k, \quad i,k = 1,3 \tag{1.1}$$

the equations of electrostatics

$$\operatorname{div} \mathbf{D} = 0, \quad \mathbf{E} = -\operatorname{grad} \varphi \tag{1.2}$$

and the material equations

$$\begin{aligned} &\sigma_{11} = c_{11}\partial_1 u_1 + c_{13}\partial_3 u_3 - e_{31}E_3, \quad \sigma_{13} = c_{44}(\partial_1 u_3 + \partial_3 u_1) - e_{15}E_1 \\ &\sigma_{33} = c_{13}\partial_1 u_1 + c_{33}\partial_3 u_3 - e_{33}E_3 \\ &D_1 = \varepsilon_{11}E_1 + e_{15}(\partial_1 u_3 + \partial_3 u_1), \quad D_3 = \varepsilon_{33}E_3 + e_{31}\partial_1 u_1 + e_{33}\partial_3 u_3 \end{aligned} \tag{1.3}$$

Here σ_{ik}, u_i, E_i, D_i and φ and the stress tensor, the displacements, the electric field strength, the electric displacement and the electric potential, $c_{ij} = c_{ij}^E$ are the moduli of elasticity, measured for a constant electric field, $\varepsilon_{ij} = \varepsilon_{ij}^s$ are the permittivities, determined for constant deformation, e_{kj} are the piezoelectric moduli and ρ is the density of the material.

Assuming

$$u_j = \operatorname{Re}(U_j e^{-i\omega t}), \quad \sigma_{jk} = \operatorname{Re}(T_{jk} e^{-i\omega t}),$$
$$\varphi = \operatorname{Re}(\Phi e^{-i\omega t})$$

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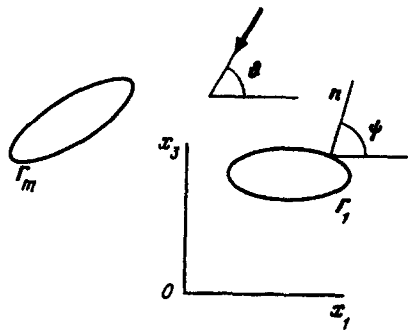

Fig. 1.

we write the system of equations (1.1)–(1.3) in matrix form in terms of the amplitudes of the displacements and the electric potential

$$\begin{Vmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{Vmatrix} \begin{Vmatrix} U_1^{(1)} & U_1^{(2)} & U_1^{(3)} \\ U_3^{(1)} & U_3^{(2)} & U_3^{(3)} \\ \Phi^{(1)} & \Phi^{(2)} & \Phi^{(3)} \end{Vmatrix} = \begin{Vmatrix} 0 \\ 0 \\ 0 \end{Vmatrix} \tag{1.4}$$

where the differential operators L_{ij} are defined as follows:

$$L_{11} = c_{11}\partial_1^2 + c_{44}\partial_3^2 + \rho\omega^2, \quad L_{12} = L_{21} = (c_{13} + c_{44})\partial_1\partial_3$$

$$L_{13} = L_{31} = (e_{31} + e_{15})\partial_1\partial_3, \quad L_{22} = c_{44}\partial_1^2 + c_{33}\partial_3^2 + \rho\omega^2$$

$$L_{23} = L_{32} = e_{15}\partial_1^2 + e_{33}\partial_3^2, \quad L_{33} = -\varepsilon_{11}\partial_1^2 - \varepsilon_{33}\partial_3^2, \quad L_{ij} = L_{ji}$$

It is necessary to supplement system (1.4) with the mechanical and electrical boundary conditions on the cavity surfaces. Assuming that the cavity surfaces border on a vacuum (air), we can assume [6]

$$D_n = D_1 \cos\psi + D_3 \sin\psi = 0 \quad \text{on} \quad \Gamma_m \tag{1.5}$$

(ψ is the angle between the normal to the contour Γ and the Ox_1 axis (Fig. 1)).

2. To construct the correct integral representations of the amplitudes of the displacements and the potential, for the purpose of reducing the initial boundary-value problem to a system of integral equations, we will use the matrix of the fundamental solutions of system (1.4) [4], which, in different notation, we will represent in the form

$$W_j^{(k)}(r,\beta,\omega) = \frac{\varepsilon_k R_k}{2\pi^2 c_{44}^3} \int_0^\pi \left\{ \sum_{\nu=1}^{2} \lambda_{\nu j}^{(k)} \Psi(\theta_\nu) + \delta_j^3 \delta_k^3 \lambda_0 \Omega(\theta) \right\} \frac{d\alpha}{A(\alpha)} \tag{2.1}$$

$$W_1^{(k)} = U_1^{(k)}, \quad W_2^{(k)} = U_3^{(k)}, \quad W_3^{(k)} = \Phi^{(k)}, \quad k = 1,2,3$$

Here

$$\Psi(x) = \frac{\pi i}{2} e^{ix} - \cos x \operatorname{ci} x - \sin x \operatorname{si} x, \quad \Omega(x) = -\ln x - C$$

$$\theta_\nu = \gamma\rho_\nu r|\cos(\alpha - \beta)|, \quad \theta = r|\cos(\alpha - \beta)|, \quad j,k = 1,2,3$$

$$\lambda_{\nu j}^{(k)} = \frac{\rho_\nu^2 b_j^{(k)} + c_j^{(k)} + \delta_j^3 \delta_k^3 c_{44}^2 / \rho_\nu^2}{(-1)^\nu (\rho_2^2 - \rho_1^2)}, \quad \lambda_0 = \frac{c_{44}^2}{\rho_1^2 \rho_2^2}$$

$$A(\alpha) = c_{44}^{-3}(a_{11}a_{22}a_{33} + 2a_{12}a_{13}a_{23} - a_{11}a_{23}^2 - a_{22}a_{13}^2 - a_{33}a_{12}^2)$$

$$A_1(\alpha)=(a_{11}a_{33}+a_{22}a_{33}-a_{23}^2-a_{13}^2)/c_{44}^2,\quad A_2(\alpha)=a_{33}/c_{44}$$

$$a_{11}=-c_{11}n_1^2-c_{44}n_3^2,\quad a_{13}=-(e_{31}+e_{15})n_1n_3$$

$$a_{12}=-(c_{13}+c_{44})n_1n_3,\quad a_{22}=-c_{44}n_1^2-c_{33}n_3^2,\quad n_1=\cos\alpha$$

$$a_{23}=-e_{15}n_1^2-e_{33}n_3^2,\quad a_{33}=\varepsilon_{11}n_1^2+\varepsilon_{33}n_3^2,\quad n_3=\sin\alpha$$

$$b_1^{(1)}=a_{22}a_{33}-a_{23}^2,\quad b_2^{(1)}=a_{13}a_{23}-a_{12}a_{33},\quad b_3^{(1)}=a_{12}a_{23}-a_{13}a_{22}$$

$$b_2^{(2)}=a_{11}a_{33}-a_{13}^2,\quad b_3^{(2)}=a_{12}a_{13}-a_{11}a_{23},\quad b_3^{(3)}=a_{11}a_{22}-a_{12}^2$$

$$c_1^{(1)}=c_{44}a_{33},\quad c_3^{(1)}=-c_{44}a_{13},\quad c_2^{(1)}=0,\quad c_2^{(2)}=c_{44}a_{33}$$

$$c_3^{(2)}=-c_{44}a_{23},\quad c_3^{(3)}=c_{44}(a_{11}+a_{22}),\quad c_k^{(j)}=c_j^{(k)},\quad b_k^{(j)}=b_j^{(k)}$$

$$\rho_\nu^2(\alpha)=-\frac{B_1}{2}+(-1)^\nu\sqrt{\frac{B_1^2}{4}-B_2},\quad \nu=1,2$$

$$B_i(\alpha)=\frac{A_i(\alpha)}{A(\alpha)},\quad \gamma=\frac{\omega}{c},\quad c=\sqrt{\frac{c_{44}}{\rho}}$$

$$R_1=P_1,\quad R_2=P_3,\quad R_3=Q,\quad \varepsilon_1=\varepsilon_2=-1,\quad \varepsilon_3=1$$

Note that the functions (2.1) are the solution of the problem of the harmonic excitation of a piezoceramic medium acted upon by mechanical forces $X_k(x_1, x_3, t) = \delta(x_1, x_3)\,\mathrm{Re}(P_k e^{-i\omega t})$ $(k = 1, 3)$ concentrated along the line $x_1 = 0$, $-\infty < x_2 < \infty$, $x_3 = 0$, or electric charges $q(x_1, x_3, t) = \delta(x_1, x_3)$ $\mathrm{Re}(Qe^{-i\omega t})$, where $\delta(x_1, x_3)$ is the Dirac delta function. The superscript $k = 1, 2, 3$ in (2.1) indicates correspondence between the quantities $U_j^{(k)}$ and $\Phi^{(k)}$ and the concentrated loads X_1, X_3 and q, δ_m^i is the Kronecker delta, $C = 0.5772$ is Euler's constant, $\mathrm{si}\,x$, $\mathrm{ci}\,x$ are the integral signs and cosines [7], and (r, β) are the polar coordinates of a point in the x_1Ox_3 plane.

It follows from an analysis of the system of differential equations (1.4) that two types of plane monochromatic waves, namely, quasi-longitudinal and quasi-transverse waves, can exist in a piezoceramic medium [6]. The velocities of these waves depend on their direction of propagation. Here the amplitudes of the displacements and the electric potential corresponding to a wave with wave number γ_ν, are given by the expressions

$$W_{j\nu}^0=\tau_j\exp[-i\gamma_\nu(\vartheta)(x_1\cos\vartheta+x_3\sin\vartheta)]\quad(\nu=1,2;\ j=1,2,3)$$
$$\gamma_\nu(\vartheta)=\frac{\omega}{c_\nu(\vartheta)},\quad c_\nu(\vartheta)=\frac{\sqrt{c_{44}}}{\rho_\nu(\vartheta)\sqrt{\rho}} \tag{2.2}$$

Here ϑ is the angle between the normal to the wave front and the x_1 axis, and the functions $\rho_\nu(\vartheta)$ are defined in (2.1). The amplitude of the potential $W_{3\nu}^0$ in each of the acoustoelectric waves is related to the amplitudes of the displacements $W_{1\nu}^0$ and $W_{2\nu}^0$ as follows:

$$\tau_3=\frac{\tfrac{1}{2}\tau_1(e_{15}+e_{31})\sin 2\vartheta+\tau_2(e_{15}\cos^2\vartheta+e_{33}\sin^2\vartheta)}{\varepsilon_{11}\cos^2\vartheta+\varepsilon_{33}\sin^2\vartheta} \tag{2.3}$$

3. The overall acoustoelectric field in a piezoelectric medium with defects is made up of the fields due to radiated waves, and the fields scattered by the cavities. Using (2.1), we can represent the amplitudes of the displacements and of the electric potential, corresponding to the scattered field, in the form

$$W_j(z)=\sum_{k=1}^{3}\int_\Gamma p_k(\zeta)g_{kj}(\zeta,z)ds \tag{3.1}$$

$$g_{kj}(\zeta,z)=\frac{\varepsilon_k}{2\pi^2c_{44}^3}\int_0^\pi\left\{\sum_{\nu=1}^{2}\lambda_{\nu j}^{(k)}\Psi(\theta_\nu^*)+\delta_j^3\delta_k^3\lambda_0\Omega(\theta^*)\right\}\frac{d\alpha}{A(\alpha)}$$

$$\theta_\nu^*=\gamma\rho_\nu r^*|\cos(\alpha-\beta^*)|,\quad \theta^*=r^*|\cos(\alpha-\beta^*)|$$

$$r^*=|\zeta-z|,\quad \beta^*=\arg(\zeta-z),\quad z=x_1+ix_3,\quad \zeta\in\Gamma$$

Here $p_k(\zeta)$ are unknown densities, to be determined, and ds is an element of the arc of the contour Γ. The integral representations (3.1) possess the necessary completeness in relation to the boundary-value problem considered.

Differentiating (3.1), we obtain

$$\partial_l W_j(z) = \sum_{k=1}^{3} \int_\Gamma p_k(\zeta) G_{kj}^{(l)}(\zeta, z)ds, \quad l = 1,3 \tag{3.2}$$

$$G_{kj}^{(l)}(\zeta,z) = \frac{\varepsilon_k}{2\pi^2 c_{44}^3} \int_0^\pi \left\{ \sum_{\nu=1}^{2} \lambda_{\nu j}^{(k)} \Psi^{(l)}(\theta_\nu^*) + \delta_j^3 \delta_k^3 \lambda_0 \Omega^{(l)}(\theta^*) \right\} \frac{d\alpha}{A(\alpha)}$$

$$\Psi^{(l)}(\theta_\nu^*) = -\gamma \rho_\nu n_l H(\theta_\nu^*) \operatorname{sign}\cos(\alpha - \beta^*), \quad \Omega^{(l)}(\theta^*) = -\frac{n_l}{r^* \cos(\alpha - \beta^*)}$$

$$H(x) = \frac{\pi}{2} e^{ix} + \frac{1}{x} + \cos x \operatorname{si} x - \sin x \operatorname{ci} x$$

Taking the functions (3.2) and the material equations (1.2) into account, we can find expressions for the amplitudes of the stresses and the components of the induction and electric-field vectors at any point of the region $z \notin \Gamma$. When $z \to \zeta_0 \in \Gamma$, when calculating the quantity $\partial_l W_j(z)$, one needs to take into account the terms outside the integrals, which arise due to the singular nature of the kernels $G_{kj}^{(l)}(\zeta, z)$ at the point $z = \zeta$.

It can be shown that the singular terms occurring in the functions $G_{kj}^{(l)}(\zeta, z)$, correspond to static loading of the piezoelectric medium. Using (2.1) we obtain, after reduction

$$\partial_l (W_j^{(k)} - W_{j0}^{(k)}) = \frac{\varepsilon_k R_k}{2\pi^2 c_{44}^3} \sum_{\nu=1}^{2} \int_0^\pi \lambda_{\nu j}^{(k)} \Psi_*^{(l)}(\theta_\nu) \frac{d\alpha}{A(\alpha)} \tag{3.3}$$

$$\Psi_*^{(l)}(\theta_\nu) = -\gamma \rho_\nu n_l h(\theta_\nu) \operatorname{sign}\cos(\alpha - \beta), \quad h(x) = H(x) - 1/x$$

where $W_{j0}^{(k)}$ are the static values of the displacements and of the electric potential. Consequently, the function (3.3) vanishes when $z = \zeta$, i.e. is regular.

A fairly lengthy procedure for the analytic evaluation of the integrals, corresponding to the static part of the derivatives of the fundamental solution (2.1), leads to the following result

$$\partial_l W_{j0}^{(k)} = -\frac{\varepsilon_k R_k}{2\pi^2 c_{44}^3 r} \int_0^\pi \frac{b_j^{(k)} n_l d\alpha}{A(\alpha)\cos(\alpha - \beta)} = R_k \operatorname{Re} \sum_{\nu=1}^{3} \frac{\omega_{k\nu} m_{l\nu} A_{\nu j}^{(k)}}{z_\nu - z_{\nu 0}} \tag{3.4}$$

$$A_{\nu 1}^{(1)} = -(c_{44} + c_{33}\mu_\nu^2)(\varepsilon_{11} + \varepsilon_{33}\mu_\nu^2) - (e_{15} + e_{33}\mu_\nu^2)^2$$

$$A_{\nu 2}^{(1)} = \mu_\nu[(c_{44} + c_{13})(\varepsilon_{11} + \varepsilon_{33}\mu_\nu^2) + (e_{15} + e_{31})(e_{15} + e_{33}\mu_\nu^2)]$$

$$A_{\nu 3}^{(1)} = \mu_\nu[(c_{44} + c_{13})(e_{15} + e_{33}\mu_\nu^2) - (e_{15} + e_{31})(c_{44} + c_{33}\mu_\nu^2)]$$

$$A_{\nu 2}^{(2)} = -(c_{11} + c_{44}\mu_\nu^2)(\varepsilon_{11} + \varepsilon_{33}\mu_\nu^2) - \mu_\nu^2(e_{15} + e_{31})^2$$

$$A_{\nu 3}^{(2)} = \mu_\nu^2(e_{15} + e_{31})(c_{44} + c_{13}) - (e_{15} + e_{33}\mu_\nu^2)(c_{11} + c_{44}\mu_\nu^2)$$

$$A_{\nu 3}^{(3)} = (c_{11} + c_{44}\mu_\nu^2)(c_{44} + c_{33}\mu_\nu^2) - \mu_\nu^2(c_{31} + c_{44})^2, \quad A_{\nu i}^{(k)} = A_{\nu k}^{(i)}$$

$$m_{1\nu} = 1, \quad m_{3\nu} = \mu_\nu, \quad z_\nu = x_1 + \mu_\nu x_3, \quad z_{\nu 0} = x_{10} + \mu_\nu x_{30}$$

where $\mu_\nu(\operatorname{Im}\mu_\nu > 0, \nu = 1, 2, 3)$ are the roots of the algebraic equation

$$a\mu^6 + b\mu^4 + c\mu^2 + d = 0$$

$$a = -c_{44}c_{33}\varepsilon_{33}(1 + k_{33}^2), \quad k_{33}^2 = \frac{e_{33}^2}{c_{33}\varepsilon_{33}}, \quad k_{15}^2 = \frac{e_{15}^2}{c_{44}\varepsilon_{11}}$$

$$b = c_{44}(2c_{13}\varepsilon_{33} - c_{33}\varepsilon_{11} + 2e_{31}e_{33}) - c_{11}c_{33}\varepsilon_{33}(1 + k_{33}^2) - c_{33}(e_{15} + e_{31})^2 +$$

$$+2e_{33}c_{13}(e_{15} + e_{31}) + c_{13}^2\varepsilon_{33}$$

$$c = c_{44}(2c_{13}\varepsilon_{11} - e_{31}^2) + c_{13}^2\varepsilon_{11} + 2c_{13}e_{15}(e_{15} + e_{31}) - c_{11}(c_{44}\varepsilon_{33} + c_{33}\varepsilon_{11} + 2e_{15}e_{33}),$$

$$d = -c_{11}c_{44}\varepsilon_{11}(1 + k_{15}^2)$$

The constants $\omega_{k\nu}$ are found from the three real systems of linear algebraic equations

$$\operatorname{Im}\sum_{\nu=1}^{3} d_{\nu j}^{(k)}\omega_{k\nu} = f_j^{(k)}, \quad j=1,2,\ldots,6; \quad k=1,2,3 \tag{3.5}$$

$$d_{\nu 1}^{(k)} = -\gamma_\nu^{(k)}\mu_\nu, \quad d_{\nu 2}^{(k)} = \gamma_\nu^{(k)}, \quad d_{\nu 3}^{(k)} = -r_\nu^{(k)}, \quad d_{\nu,i+3}^{(k)} = -A_{\nu i}^{(k)}$$

$$f_i^{(k)} = \delta_i^k / 2\pi, \quad f_{i+3}^{(k)} = 0, \quad i=1,2,3$$

$$\gamma_\nu^{(k)} = c_{13}A_{\nu 1}^{(k)} + (c_{33}A_{\nu 2}^{(k)} + e_{33}A_{\nu 3}^{(k)})\mu_\nu$$

$$r_\nu^{(k)} = (\varepsilon_{33}A_{\nu 3}^{(k)} - e_{33}A_{\nu 2}^{(k)})\mu_\nu - e_{31}A_{\nu 1}^{(k)}$$

Hence, using (3.3) and (3.4) the integral representations (3.2) can be written in the form

$$\partial_l W_j(z) = \sum_{k=1}^{3}\int_\Gamma p_k(\zeta)F_{kj}^{(l)}(\zeta,z)ds, \quad l=1,3 \tag{3.6}$$

$$F_{kj}^{(l)}(\zeta,z) = \operatorname{Re}\sum_{\nu=1}^{3}\frac{\omega_{k\nu}m_{l\nu}A_{\nu j}^{(k)}}{z_\nu-\zeta_\nu} + \frac{\varepsilon_k}{2\pi^2 c_{44}^3}\sum_{\nu=1}^{2}\int_0^\pi \lambda_{\nu j}^{(k)}\Psi_*^{(l)}(\theta_\nu^*)\frac{d\alpha}{A(\alpha)}$$

Substituting the limiting values of the derivatives (3.6) as $z\to\zeta_0\in\Gamma$ into the mechanical and electrical boundary conditions, we arrive at a system of three singular integral equations of the second kind in the functions $p_k(\zeta)$

$$\frac{1}{2}p_l(\zeta_0) + \sum_{k=1}^{3}\int_\Gamma p_k(\zeta)M_{lk}(\zeta,\zeta_0)ds = N_l(\zeta_0), \quad l=1,2,3 \tag{3.7}$$

$$M_{1k} = T_{11}^{(k)}n_1^* + T_{13}^{(k)}n_3^*, \quad M_{2k} = T_{13}^{(k)}n_1^* + T_{33}^{(k)}n_3^*$$

$$M_{3k} = [e_{15}(F_{0k2}^{(1)} + F_{0k1}^{(3)}) - \varepsilon_{11}F_{0k3}^{(1)}]n_1^* + (e_{31}F_{0k1}^{(1)} + e_{33}F_{0k2}^{(3)} - \varepsilon_{33}F_{0k3}^{(3)})n_3^*,$$

$$F_{0kj}^{(l)} = F_{kj}^{(l)}(\zeta,\zeta_0)$$

$$T_{11}^{(k)} = c_{11}F_{0k1}^{(1)} + c_{13}F_{0k2}^{(3)} + e_{31}F_{0k3}^{(3)}$$

$$T_{13}^{(k)} = c_{44}(F_{0k2}^{(1)} + cF_{0k1}^{(3)}) + e_{15}F_{0k3}^{(1)}$$

$$T_{33}^{(k)} = c_{13}F_{0k1}^{(1)} + c_{33}F_{0k2}^{(3)} + e_{33}F_{0k3}^{(3)}$$

$$N_l(\zeta_0) = \sum_{j=1}^{2}N_l^{(j)}(\zeta_0), \quad n_1^* = \cos\psi_0, \quad n_3^* = \sin\psi_0$$

$$N_1^{(1)}(\zeta_0) = c_{13}\left(1+\frac{e_{31}e_{33}}{\varepsilon_{33}c_{13}}\right)n_1^*\chi_1, \quad N_2^{(1)}(\zeta_0) = c_{33}(1+k_{33}^2)n_3^*\chi_1$$

$$N_1^{(2)}(\zeta_0) = c_{44}(1+k_{15}^2)n_3^*\chi_2, \quad N_2^{(2)}(\zeta_0) = c_{44}(1+k_{15}^2)n_1^*\chi_2$$

$$\chi_1 = \tau_2 i\gamma_1\left(\frac{\pi}{2}\right)\exp\left(-i\gamma_1\left(\frac{\pi}{2}\right)\operatorname{Im}\zeta_0\right), \quad \chi_2 = \tau_2 i\gamma_2(0)\exp(-i\gamma_2(0)\operatorname{Re}\zeta_0)$$

$$\psi_0 = \psi(\zeta_0); \quad \zeta,\zeta_0\in\Gamma = \cup\Gamma_m, \quad m=1,2,\ldots,n; \quad j=1,2$$

The right-hand sides of Eqs (3.7) $N_l^{(j)}$ (ζ_0) were found using expressions (2.2) and (2.3); they correspond to two types of loadings: a plane quasi-longitudinal wave propagating in a negative direction of the x_3 axis (j = 1) and a plane quasi-transverse wave, radiated in the negative direction of the x_1 axis (j = 2). The third integral equation (l = 3) in system (3.7) corresponds to electrical boundary condition (1.5).

4. To determine the dynamic stress concentration in the piezoceramic medium with cavities we will calculate the normal stress σ_θ on the contour Γ. Bearing the integral representations (3.6) in mind, we obtain

$$\sigma_\theta = \mathrm{Re}(T_\theta e^{-i\omega t}), \quad T_\theta(\zeta_0) = T_0 + \sum_{k=1}^{3} \left\{ p_k(\zeta_0) t_k(\zeta_0) + \int_\Gamma p_k(\zeta) T_\theta^{(k)}(\zeta, \zeta_0) ds \right\}$$

$$t_k(\zeta_0) = -\pi \,\mathrm{Im} \sum_{\nu=1}^{3} \frac{(\mu_\nu n_3^* + n_1^*)^2}{\mu_\nu n_1^* - n_3^*} \omega_{k\nu} \gamma_\nu^{(k)} \tag{4.1}$$

$$T_\theta^{(k)} = T_{11}^{(k)} (n_3^*)^2 + T_{33}^{(k)} (n_1^*)^2 - 2T_{13}^{(k)} n_1^* n_3^*$$

$$T_0 = -\left[\left(c_{13} + \frac{e_{31} e_{33}}{\varepsilon_{33}} \right) (n_3^*)^2 + c_{33}(1 + k_{33}^2)(n_1^*)^2 \right] \chi_1 + 2c_{44}(1 + k_{15}^2) n_1^* n_3^* \chi_2$$

The quantities $T_{ij}^{(k)}$ occurring here were defined in (3.7).

We will consider, as an example, a piezoelectric medium (PZT-4 ceramics [8]), weakened by a cavity of circular or square cross-section with the parametric equations

$$\mathrm{Re}\zeta = a(\cos\eta + c\cos 3\eta), \quad \mathrm{Im}\zeta = a(\sin\eta - c\sin 3\eta), \quad \eta \in [0, 2\pi] \tag{4.2}$$

Here $c = 0$ for a cavity of circular cross section and $c = 0.14036$ for a cavity of square cross-section.

The functions $p_k(\zeta)(k = 1, 2, 3)$ were calculated from (3.7) using (4.3) by the method of quadratures [9], and then, using (4.1), we determined the stress $T_\theta(\zeta_0)$. For PZT-4 ceramics we have μ_k.

Figure 2 illustrates the change in the quantity $\chi = | T_\theta/\Lambda |$ at the points $\eta = \pi$ (curve 1) and $\eta = 3\pi/2$ (curves 2 and 3) of the contour of a circular cavity as a function of the normalized wave number γa for a quasi-longitudinal wave (curves 1 and 3) or a quasi-transverse wave (curve 2). The dashed curves are drawn for values of the piezoelectric moduli $e_{33} = e_{13} = 0$ and $e_{15} = 0.1$ C/m^2, which corresponds in practice to a piezoelectrically passive material (for all $e_{ij} = 0$ system (3.5) is degenerate). In the first case of loading, the quantity $\Lambda = | T_{33}^0 | = \tau_2\gamma\sqrt{(c_{44}c_{33}(1 + k_{33}^2))}$ is the modulus of the amplitude of the stress σ_{33} in the quasi-longitudinal wave, and in the second case $\Lambda = | T_{33}^0 | = \tau_2\gamma c_{44}\sqrt{(1 + k_{15}^2)}$ is the modulus of the amplitude of the stress σ_{13} in the quasi-transverse wave.

The graphs of the change in the value of χ at the points $\eta = 3\pi/2$ and $\eta = 157\pi/158$ of the contour of the square cavity are shown in Fig. 3. Curves 1–3 correspond to those in Fig. 2.

The distribution of the quantity χ on the contour of the square opening when quasi-longitudinal and quasi-transverse waves are radiated is shown in Fig. 4. The curve with number m corresponds to a value of the normalized wave number $\gamma a = m$.

It follows from these results that, in dynamic excitation, a redistribution of the stress σ_θ over the cavity surface occurs. The influence of the inertial effect manifests itself in an increase in σ_θ over a certain range of variation of the angular frequency compared with its static analogue. The effect of the connectedness of the acoustoelectric fields, as can be seen from Fig. 2, may make a considerable contribution to the stress concentration, which is not observed in the case of plane deformation of a piezoelectric ceramic medium in a plane perpendicular to the direction of polarization of the material.

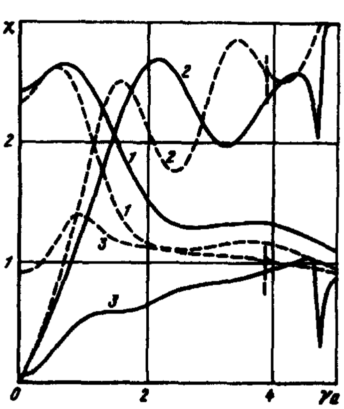

Fig. 2.

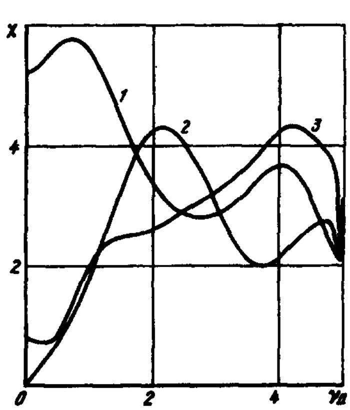

Fig. 3.

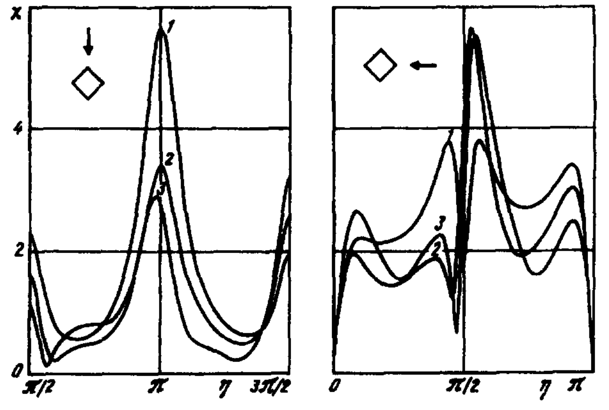

Fig. 4.

REFERENCES

1. PARTON, V. Z. and FIL'SHTINSKII, M. L., The dynamic problem of the theory of elasticity for a piezoelectric medium with tunnel cuts. *Izv. Akad. Nauk ArmSSR. Mekhanika*, 1989, **42**, 5, 17–24.
2. FIL'SHTINSKII, M. L., The unsteady dynamic problem of electroelasticity for an inorganic medium with curvilinear tunnel cracks. *Prikl. Mat. Mekh.*, 1994, **58**, 4, 159–166.
3. FIL'SHTINSKII, M. L., Harmonic oscillations of a piezoelectric half-space with tunnel cavities (longitudinal shear deformation). *Aukust. Zh.*, 1991, **37**, 4, 777–781.
4. VATUL'YAN, A .O. and KUBLIKOV, V. L., Boundary integral equations in electroelasticity. *Prikl. Mat. Mekh.*, 1989, **53**, 6, 1037–1041.
5. MUSKHELISHVILI, N. I., *Singular Integral Equations*. Nauka, Moscow, 1968.
6. PARTON, V. Z. and KUDRYAVTSEV, B. A., *Electromagnetoelasticity of Piezoelectric and Electrically Conducting Solids*. Nauka, Moscow, 1988.
7. LUKE, Yu. L., *Mathematical Functions and Their Approximations*. Academic Press, New York, 1975.
8. BERLINCOURT, D. A., CURRAN, D. R. and JAFFE, H., Piezoelectric and piezomagnetic materials and their functions as transducers. In *Physical Acoustics*, Vol. 1, Part A. Edited by W. P. Mason. Academic Press, New York, 1964.
9. PANASYUK, V. V., SAVRUK, M. P. and NAZARCHUK, Z. T., *The Method of Singular Integral Equations in Two-dimensional Diffraction Problems*. Naukova Dumka, Kiev, 1984.

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